CORNELL NOTES

TOPIC/OBJECTIVE:

CONTENT/CLASS:

NAME:

CLASS/PERIOD:

DATE:

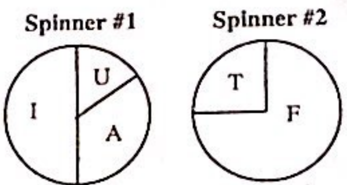
ESSENTIAL QUESTION:

QUESTIONS:

NOTES:

Probability Models

When all the possible outcomes of a probabilistic event have the same probability, probabilities can be calculated by listing the possible outcomes in a **systematic list**. However, when some outcomes are more probable than others, a more sophisticated model is required to calculate probabilities.



$P(I) = \frac{1}{2}$ $P(T) = \frac{1}{4}$

$P(U) = \frac{1}{6}$ $P(F) = \frac{3}{4}$

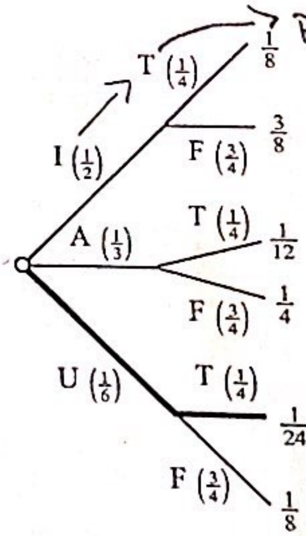
$P(A) = \frac{1}{3}$

One such model is an **area model**. In this type of model, the situation is represented by a square with area of 1 so that the areas of the parts are the probabilities of the different events that occur. For example, suppose you spin the two spinners shown above. The possible outcomes are represented in the area model at right. Notice that column "U" takes up $\frac{1}{6}$ of the width of the table since the "U" region is $\frac{1}{6}$ of Spinner #1. Similarly, the "T" row takes up $\frac{1}{4}$ of the height of the table, since the "T" region is $\frac{1}{4}$ of Spinner #2. Then the probability that the spinners come out "U" and "T" is equal to the area of the "UT" rectangle in the table: $\frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24}$.

Spinner #1

Spinner #2	I $(\frac{1}{2})$	A $(\frac{1}{3})$	U $(\frac{1}{6})$
T $(\frac{1}{4})$	P(IT) $= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$	P(AT) $= \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$	P(UT) $= \frac{1}{4} \cdot \frac{1}{6} = \frac{1}{24}$
F $(\frac{3}{4})$	P(IF) $= \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$	P(AF) $= \frac{1}{3} \cdot \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$	P(UF) $= \frac{1}{6} \cdot \frac{3}{4} = \frac{3}{24} = \frac{1}{8}$

The situation can also be represented using a **tree diagram**. In this model, the branching points indicate probabilistic events, and the branches stemming from each event indicate the possible outcomes for the event. For example, in the tree diagram at right the first branching point represents spinning the first spinner. The first spinner can come out "I" "A" or "U", so each of those options has a branch. The numbers next to each letter represent the probability that that letter occurs.



$P(IT) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

The numbers at the far right of the table represent the probabilities of various outcomes. For example, the probability of spinning "U" and "T" can be found at the end of the bold branch of the tree. This probability, $\frac{1}{24}$, can be found by multiplying the fractions that appear on the bold branches.

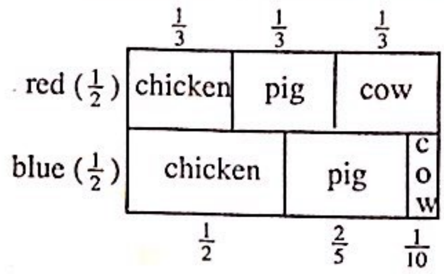
SUMMARY:

QUESTIONS:

NOTES:

Conditional Probability

When you are calculating a probability, but have been given additional information about what has already occurred, you are calculating a **conditional probability**. In problem 9-47 (Build-a-Farm) you computed the conditional probability that you spun red given that you knew you had already gotten a cow counter. The probabilities in that problem were as follows:



"given"

$P(R|C)$ is shorthand notation for for "the probability of a red spin given that you got a cow." Using the modified counting rule,

$$P(R\,|\,C)=\frac{P(\text{red occuring within the cows})}{P(C)}=\frac{\frac{1}{6}}{\frac{13}{60}}=\frac{10}{13}$$

In general, the probability of an event B, given that event A has already occurred is given by the **multiplication rule**:

$$P(B|A)=\frac{P(B\text{ and }A)}{P(A)}$$

MATH NOTES

Mutually Exclusive

Mutually exclusive events or disjoint events—they mean the same thing—can never both happen at the same time. When one occurs, it means the other cannot possibly occur. If event B occurs, we know that event A cannot occur: $P(A \cap B)=0$. So the occurrence of B tells us about the probability of A occurring. When events are mutually exclusive they are completely dependent on each other: mutually exclusive events cannot be independent. Mutually exclusive events are not very interesting in statistics.

Being naturally blonde and having naturally black hair are mutually exclusive; if one occurs, the other cannot possible occur. If your friend tells you the next person has naturally black hair, you are 100% certain they do not have naturally blonde hair. So the probability of blonde has changed, knowing that the person has black hair. They are not independent: $P(\text{blonde}\,|\,\text{black}) \neq P(\text{blonde})$.

SUMMARY: